Physics 5C - Second Midterm

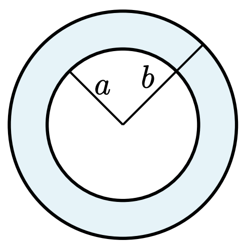
Wednesday, May 21, 9-9:50AM

UCLA / Spring 2025 / Brian Naranjo

Solutions

1) (25 points) Consider a copper tube of electrical resistivity ρ, inner radius a, outer radius b, and length c. Treating the tube's ends as terminals, we wire the tube to a battery of emf $\mathcal{E}_0$, so that current flows along the tube's axial direction. Find the resulting power dissipation P_0.

Solution



The tube's resistance between its terminals is

$$R = \frac{\rho L}{A} = \frac{\rho c}{\pi(b^2 - a^2)}.$$

The power dissipation in the resistor is then

$$\boxed{P_0 = \frac{\mathcal{E}_0^2}{R} = \frac{\mathcal{E}_0^2 \pi(b^2 - a^2)}{\rho c}}$$

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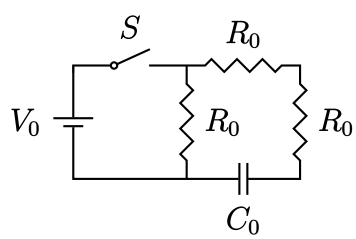
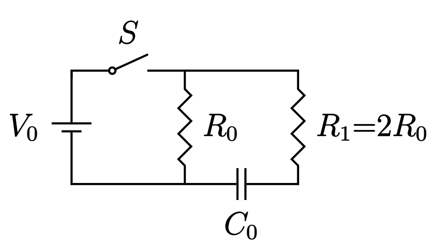
2) (25 points) Initially, switch S is open and capacitor C_0 is uncharged. At time $t = 0$, the switch is closed. Find the subsequent *total* power dissipated in all three resistors, $P_R(t)$, as a function of time.



Solution The two resistors on the circuit's upper and right sides are in series. Replacing them with a single equivalent resistor, the fully simplified circuit is



Once the switch is closed, the potential across resistor R_0 is equal to V_0 for all time because it is wired in parallel with the battery. Therefore, from Ohm's law, the current through R_0 is equal to $I_0 = V_0/R_0$, and, for all time, the power dissipated in R_0 is

$$P_0 = \frac{V_0^2}{R_0}.$$

The current $I_1(t)$ through resistor R_1, on the other hand, starts off with a finite value $I_1(t=0) = V_0/2R_0$, but then decays to zero with RC time constant $\tau = 2R_0C_0$. As usual, we adopt the general solution to our given boundary conditions. We find the *time-dependent* current and power dissipated in the equivalent resistor,

$$I_1(t) = \frac{V_0}{2R_0}e^{-t/\tau} \quad \text{and} \quad P_1(t) = I_1^2(t)R_1 = \frac{V_0^2}{2R_0}e^{-2t/\tau}$$

Thus, the total power dissipated in all the circuit's resistors is

$$\boxed{P_R(t) = P_0 + P_1(t) = \frac{V_0^2}{R_0}\left[1 + \frac{1}{2}e^{-t/(R_0C_0)}\right]}$$

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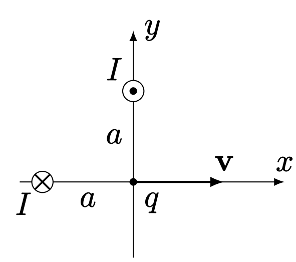
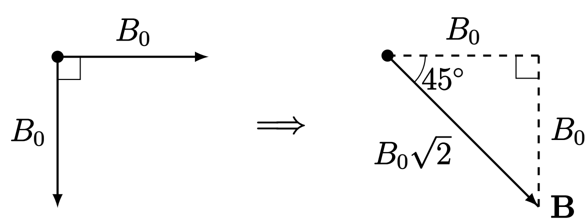
3) (25 points) An infinite straight-line current, directed out-of-the-page, intercepts the positive y-axis at a distance a from the origin. A second infinite straight-line current, directed into-the-page, intercepts the negative x-axis at a distance a from the origin. Both currents have magnitude I. A positive point charge $q > 0$, located at the origin, moves with velocity $\mathbf{v}$ in the positive x-direction. Find both the magnitude and direction of the magnetic force $\mathbf{F}$ acting on the point charge.



Solution We first find the net magnetic field at the origin. Both currents are the same distance a from the origin, so both their contributions to the magnetic field at the origin have the same magnitude,

$$B_0 = \frac{\mu_0 I}{2\pi a}.$$

The right-hand rule states that the current coming out of the page produces a counterclockwise magnetic field in the plane. At the origin, this magnetic field points in the positive x-direction. Similarly, the current going into the page produces a clockwise magnetic field. At the origin, this magnetic field points in the negative y-direction. Recognizing the $45° - 45° - 90°$ triangle, the net magnetic field $\mathbf{B}$ is found,

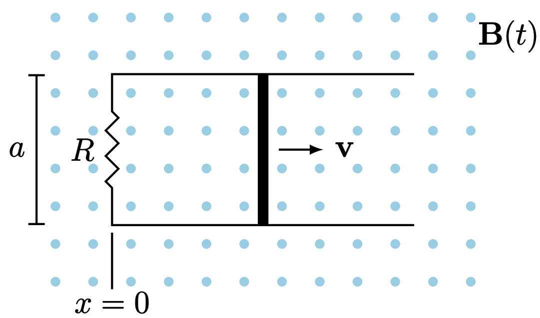


At the origin, the angle between $\mathbf{v}$ and $\mathbf{B}$ is 45°. Then, the magnitude of the force is

$$F = qvB\sin 45^\circ = qv(B_0\sqrt{2})\frac{1}{\sqrt{2}} = \boxed{\frac{qv\mu_0 I}{2\pi a}}$$

To find the direction of $\mathbf{F}$, we apply the right-hand rule from $\mathbf{v}$ toward $\mathbf{B}$, using our right thumb as a stationary axis. Our right thumb and $\mathbf{F}$ point $\boxed{\text{into the page}}$. ■

4) (25 points) A slidewire of length a is moving to the right with constant velocity $\mathbf{v}$. Its circuit has resistance R and is immersed in a magnetic field oriented out-of-the-page with a time-varying magnitude $B(t)$. Initially, at time $t = 0$, $B(0) = B_0$ and the slidewire is located at $x = x_0$. Find the subsequent magnetic field $B(t)$ such that no current is induced in the circuit for all time $t \geq 0$.



Solution Faraday's flux rule states that a time-variation in flux induces an emf,

$$\mathcal{E}_\Gamma = -\frac{\Delta\Phi}{\Delta t}.$$

If we can arrange for the flux through the circuit to remain constant, then there will be no induced emf and no current for all time $t \geq 0$. At time $t = 0$, taking loop traversal Γ to be counterclockwise, the value of the flux is the product of the initial magnetic field and the loop's initial area,

$$\Phi_0 = B_0 a x_0.$$

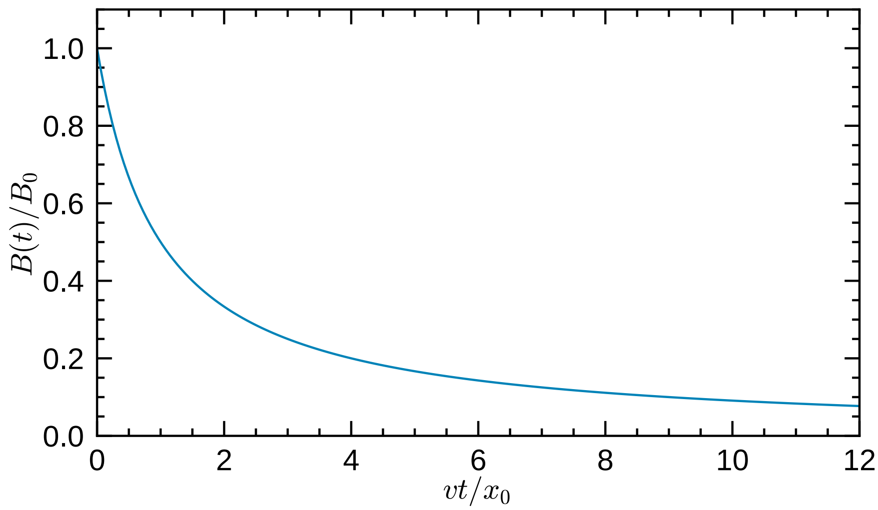
At any time t, the slidewire is located at $x(t) = x_0 + vt$. Then, we equate the initial flux to any subsequent value of the flux,

$$\Phi_0 = \Phi(t) \implies B_0 a x_0 = B(t) a (x_0 + vt) \implies \boxed{B(t) = \frac{B_0 x_0}{x_0 + vt}}$$

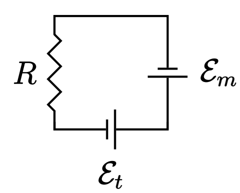
This interesting result warrants further discussion. When plotting theoretical results, it is usually preferred to plot in terms of dimensionless quantities. We rearrange our result as

$$\frac{B(t)}{B_0} = \frac{1}{1 + (vt/x_0)},$$

and then plot using dimensionless quantities $B(t)/B_0$ and $x(t)/x_0 = vt/x_0$,



As the slidewire moves to the right, through a magnetic field oriented out-of-the-page, there is a motional emf $\mathcal{E}_m$ oriented clockwise. On the other hand, at an instant in time, the magnetic field strength is decreasing, with which there must be an associated induced electric field. As the magnetic field is directed out-of-the-page, Lenz's law states that the induced electric field should be oriented counterclockwise so that its induced current would oppose this change. We sometimes refer to the emf resulting from an induced electric field as the *transformer emf* $\mathcal{E}_t$. The equivalent circuit with opposing emfs is



In this problem, we have chosen $B(t)$ such that the two emfs cancel each other out for all time $t \geq 0$. As usual, the magnitude of the motional emf is

$$\mathcal{E}_m = vB(t)a.$$

The magnitude of the transformer emf is given by the change in flux purely due to the changing magnetic field,

$$\mathcal{E}_t = ax(t)\left|\frac{dB}{dt}\right|.$$

Let's confirm that, for our solution $B(t)$, the two emfs have equal magnitude,

$$\mathcal{E}_t = ax(t)\left|\frac{dB}{dt}\right| = a(x_0 + vt)\left|\frac{B_0 x_0 v}{(x_0+vt)^2}\right| = v\left(\frac{B_0 x_0}{x_0+vt}\right)a = vB(t)a = \mathcal{E}_m.$$

We might be tempted to bring $B(t)$ abruptly down to zero right after $t = 0$. This would have terrible consequences. You would get a change in flux equal to Φ_0 in some short time interval Δt, with a resulting emf inversely proportional to Δt. Such a rapid change in flux would induce an enormous emf and current, likely evaporating the resistor. ■