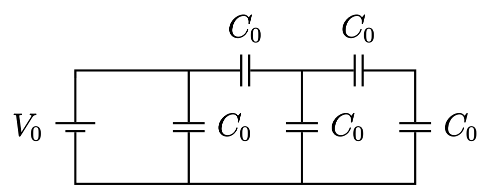
Physics 5C - First Midterm

Wednesday, April 23, 9-9:50AM

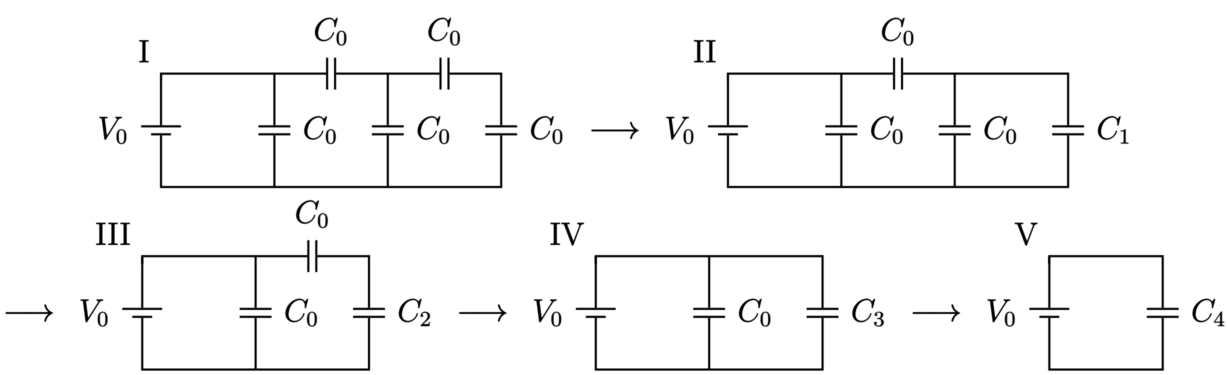
UCLA / Spring 2025 / Brian Naranjo

Solutions

1) (25 points) Five capacitors, each of capacitance C_0, and a battery of emf V_0 are connected in the circuit shown. Find the total potential energy U_{net} stored in the capacitor network.



Solution



- In Circuit I, the two rightmost capacitors C_0 and C_0 are in series,

$$C_1 = \left(\frac{1}{C_0} + \frac{1}{C_0}\right)^{-1} = \left(\frac{2}{C_0}\right)^{-1} = \frac{C_0}{2}.$$

- In Circuit II, the two rightmost capacitors C_0 and C_1 are in parallel,

$$C_2 = C_0 + C_1 = 3C_0/2.$$

- In Circuit III, the two rightmost capacitors C_0 and C_2 are in series,

$$C_3 = \left(\frac{1}{C_0} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{C_0} + \frac{2}{3C_0}\right)^{-1} = \left(\frac{5}{3C_0}\right)^{-1} = \frac{3C_0}{5}.$$

- In Circuit IV, the two remaining capacitors C_0 and C_3 are in parallel,

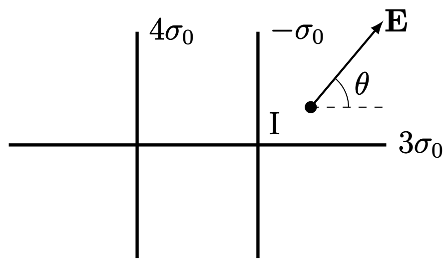
$$C_4 = C_0 + C_3 = C_0 + \frac{3C_0}{5} = \frac{8C_0}{5}$$

The net stored energy in the capacitor network equals the energy stored in its single equivalent capacitor C_4, charged to a potential difference of V_0:

$$\boxed{U_{\text{net}} = \frac{1}{2} C_4 V_0^2 = \frac{4}{5} C_0 V_0^2}$$

■

2) (25 points) Three uniformly-charged infinite planes are perpendicular to the plane of the paper, dividing space into six regions, as shown. Furthermore, the two vertical planes are perpendicular to the horizontal plane and charge density σ_0 is positive. In Region I, the electric field $\mathbf{E}$ is at an angle θ above the positive x direction. Find both the magnitude E and angle θ.



Solution The y-component of the electric field is solely due to the horizontal $3\sigma_0$ plane. As this plane is positively charged, its electric field is directed away from itself, and therefore points upward in Region I,

$$E_y = \frac{3\sigma_0}{2\epsilon_0}.$$

The x-component of the electric field is given by the superposition of the fields of the two vertical planes. In Region I, the field of the positive plane is directed towards the right and the field of the negative plane is directed towards the left. Therefore,

$$E_x = \frac{4\sigma_0}{2\epsilon_0} - \frac{\sigma_0}{2\epsilon_0} = \frac{3\sigma_0}{2\epsilon_0}.$$

The two components of the electric field have equal magnitude. Recognizing the resulting 45° – 45° – 90° triangle, we can immediately write the solution. Alternatively, the magnitude of the electric field is given by the Pythagorean theorem,

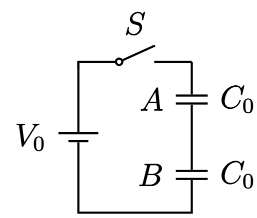
$$\boxed{E = \sqrt{E_x^2 + E_y^2} = \frac{3\sigma_0}{2\epsilon_0}\sqrt{2}},$$

and its direction is

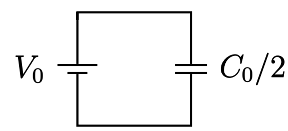
$$\boxed{\theta = \arctan\left(\frac{E_y}{E_x}\right) = \arctan 1 = 45^\circ}$$

■

3) (25 points) Initially, switch S is closed so that battery V_0 charges the two parallel-plate capacitors, each of capacitance C_0, as shown. The switch is then opened and remains open. Finally, after *doubling* the gap on capacitor A and *tripling* the gap on capacitor B, find the final potential difference across each capacitor.



Solution While charging the capacitors, with the switch closed, we combine the two identical series capacitors into a single equivalent capacitor,

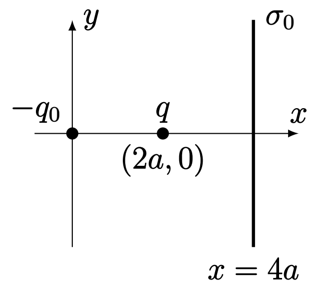


The equivalent capacitor therefore carries a net charge $Q_0 = V_0C_0/2$. Capacitors in series each carry the same charge as their equivalent, so $Q_A = Q_B = Q_0$. After the switch is opened, the battery is effectively removed from the circuit and the charge on the capacitors remains subsequently fixed. The capacitance of a parallel plate capacitor, $C = \epsilon_0 A/d$, is inversely proportional to its gap d. Therefore, after expanding the gaps, $C_A = C_0/2$ and $C_B = C_0/3$. Finally, using our fundamental capacitor relation $Q = VC$,

$$\boxed{V_A = \frac{Q_A}{C_A} = \frac{V_0C_0}{2}\frac{2}{C_0} = V_0 \quad \text{and} \quad V_B = \frac{Q_B}{C_B} = \frac{V_0C_0}{2}\frac{3}{C_0} = \frac{3}{2}V_0}$$

■

4) (25 points) An infinite plane of uniform positive surface-charge density σ_0 lies in the plane $x = 4a$, and a negative point charge $-q_0$ is fixed at the origin, as shown. A particle of mass m and positive charge q is released from rest at the point $(2a, 0)$ and moves along the x-axis. Find its speed v when it reaches the point $(a, 0)$.



Solution One way to solve this problem is by using the conservation of mechanical energy,

$$U_0 + K_0 = U_1 + K_1.$$

If $V(x)$ is the *electric potential* along the x-axis between the origin and the plane, then charge q's potential energy is $U(x) = qV(x)$. The particle starts from rest, so $K_0 = 0$. The final kinetic energy is

$$K_1 = U_0 - U_1 = qV(2a) - qV(a).$$

This quantity is equal to the work done by the electric field acting on q moving from $x = 2a$ to $x = a$.[1] The electric potential is given by the superposition of point charge $-q_0$ and plane σ_0,

$$V(x) = -\frac{kq_0}{x} + \frac{\sigma_0}{2\epsilon_0}x.$$

We confirm that the signs of both terms ensure the electric potential decreases along the direction of their respective electric fields. Both fields point to the left. Therefore, both of their contributions to $V(x)$ must slope *upwards* to the right. Then, the final kinetic energy is

$$K_1 = qV(2a) - qV(a) = q\left(-\frac{kq_0}{2a} + \frac{\sigma_0}{2\epsilon_0}2a\right) - q\left(-\frac{kq_0}{a} + \frac{\sigma_0}{2\epsilon_0}a\right) = \frac{kqq_0}{2a} + \frac{q\sigma_0 a}{2\epsilon_0}.$$

The two terms represent the work done on q by the fields of the point charge and the plane, respectively. Equating this to $K_1 = mv^2/2$ and solving for v,

$$\boxed{v = \sqrt{\frac{kqq_0}{ma} + \frac{q\sigma_0 a}{m\epsilon_0}}}$$

■

[1] The work done by the plane's electric field acting on q is simple because the force is constant, $W_{2a\to a} = Fa = [q\sigma_0/(2\epsilon_0)]a$. To treat both sources of electric field in the same way, though, we calculate $V(x)$ including contributions from both the plane with surface charge σ_0 and the point charge $-q_0$.