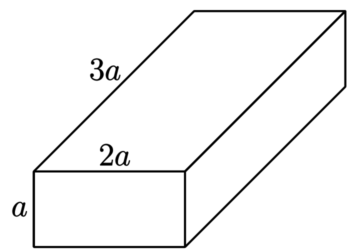
Physics 5C - Second Midterm

Wednesday, May 22, 2-2:50PM

UCLA / Spring 2024 / Brian Naranjo

Solutions

1) (25 points) A conducting material of resistivity ρ is formed into a rectangular block of side lengths a, $2a$, and $3a$, as shown below. A potential difference V is applied across one of the block's three pairs of parallel faces so that the power dissipation P is *maximized.* Find P.



Solution Consider a wire of such material with length L and cross section area A. The resistance and power dissipation of the wire is then

$$R = \frac{\rho L}{A} \quad \text{and} \quad P = \frac{V^2}{R} = \frac{V^2 A}{\rho L}.$$

Therefore, we can maximize power dissipation by maximizing the ratio A/L. We should then take the block's shortest dimension, side a, to be along the wire's length L,

$$\boxed{P_{\text{max}} = \frac{6V^2 a}{\rho}}$$

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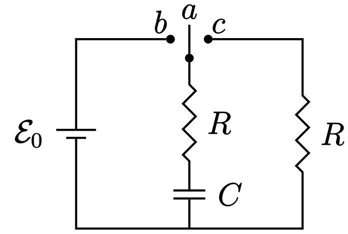
2) (25 points) The capacitor in the circuit shown below is initially uncharged and the switch is in position a. At time $t = 0$, the switch is flipped to position b. At time $t = t_0$, the switch is flipped to position c. At time $t = 2t_0$, the switch is returned to position a. At time $t = 2\,t_0$, what is the energy stored on the capacitor?



Solution During time interval $t = 0$ through $t = t_0$, the capacitor *charges* as

$$V(t) = \mathcal{E}_0 \left(1 - e^{-t/\tau}\right),$$

where $\tau = RC$. During time interval $t = t_0$ through $t = 2t_0$, the capacitor *discharges* as

$$V(t) = V(t_0)e^{-(t-t_0)/(2\tau)}.$$

The capacitor's discharging time constant is double its charging time constant because the capacitor's series resistance has now doubled from R to $2R$.

Then, the capacitor's final energy is

$$U(2t_0) = \frac{1}{2}CV^2(2t_0) = \boxed{\frac{1}{2}C\mathcal{E}_0^2\left[\left(1 - e^{-t_0/\tau}\right)e^{-t_0/(2\tau)}\right]^2 = \frac{1}{2}C\mathcal{E}_0^2\left(1 - e^{-t/\tau}\right)^2 e^{-t_0/\tau}}$$

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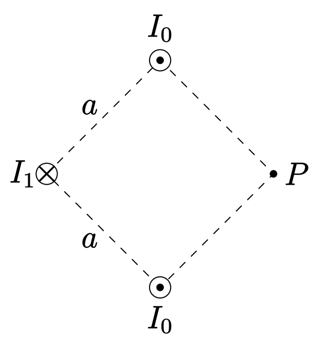
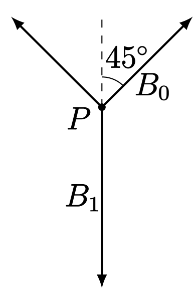
3) (25 points) Three parallel infinite line currents, each perpendicular to the page, pass through the vertices of a square of side length a, shown below. Given I_0 and a, find I_1 such that the magnetic field vanishes at observation point P.



Solution At point P, the magnetic fields of the three currents add as vectors,



By symmetry, the net magnetic field in the horizontal direction is zero. We choose the magnitude of I_1 so that the net magnetic field in the vertical direction is zero,

$$B_1 = 2B_0 \cos 45^\circ \implies \frac{\mu_0 I_1}{2\pi(\sqrt{2}a)} = 2\left(\frac{\mu_0 I_0}{2\pi a}\right)\frac{1}{\sqrt{2}} \implies \boxed{I_1 = 2I_0}$$

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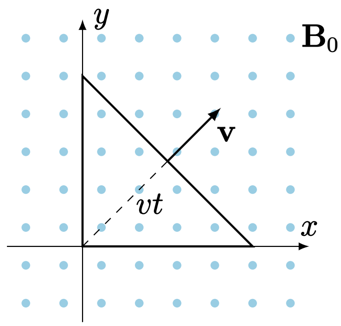
4) (25 points) A conducting wire in the shape of 45°-45°-90° triangle is in a uniform magnetic field, directed out-of-the-page with magnitude B_0. The triangle's 90° corner remains fixed at the origin, as shown. At time t, the distance between the origin and the hypotenuse is equal to vt, so that the hypotenuse is traveling with speed v. Assuming the wire's resistance per unit length is equal to λ_0, use any method to find the magnitude and direction of current $I(t)$ at time t.



Solution Recognize that this is just a tilted slidewire.

Method 1: Motional emf
The magnetic field, wire orientation, and wire velocity are mutually perpendicular. Therefore, the magnitude of the motional emf is

$$|\mathcal{E}_m| = v\ell B_0 = v(2vt)B_0 = 2v^2tB_0,$$

where $2vt$ is the length of the triangle's hypotenuse. At time t, the length of the triangle's perimeter is $2vt + 2(\sqrt{2}vt) = 2vt(1+\sqrt{2})$. Then, the magnitude of the current is

$$I = \frac{|\mathcal{E}_m|}{R} = \frac{2v^2tB_0}{\lambda_0 2vt(1+\sqrt{2})} = \boxed{\frac{vB_0}{\lambda_0(1+\sqrt{2})}}$$

From the right-hand-rule, the orientation of the motional emf, and therefore, current, is clockwise.

Method 2: Faraday's flux rule and Lenz's law
In time interval Δt, the hypotenuse, with length $\ell = 2vt$, travels a distance $\Delta s = v\Delta t$. Therefore, over the same interval, the magnetic flux increases *out-of-the-page* by an amount

$$|\Delta\Phi| = (\Delta A)B_0 = (\ell \cdot \Delta s) = (2vt \cdot v\Delta t)B_0$$

Then, Faraday's flux rule gives the magnitude of the induced emf,

$$|\mathcal{E}| = \left|\frac{\Delta\Phi}{\Delta t}\right| = 2v^2tB_0,$$

which is the same value of emf obtained using Method 1. Proceeding as before,

$$I = \frac{|\mathcal{E}|}{R} = \boxed{\frac{vB_0}{\lambda_0(1+\sqrt{2})}}$$

If we are careful about our direction of traversal, we may obtain the orientation of the emf and current. Alternatively, we may use Lenz's law. As the slidewire travels, the flux is increasing *out-of-the-page*. Therefore, the induced current will be clockwise to oppose this change. ■