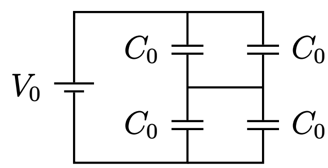
Physics 5C - First Midterm

Wednesday, April 24, 2-2:50PM

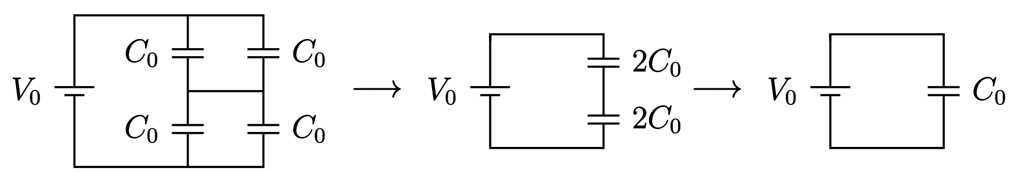
UCLA / Spring 2024 / Brian Naranjo

Solutions

1) (25 points) Four capacitors, each of capacitance C_0, and a battery of emf V_0 are connected in the circuit shown below. Find the total potential energy stored in the capacitors.



Solution



In the first diagram, we identify two pairs of parallel capacitors. Each each pair has equivalent capacitance $C_0 + C_0 = 2C_0$. In the second diagram, the two capacitors are in series, so the overall equivalent capacitance of the four capacitors is

$$\left(\frac{1}{2C_0} + \frac{1}{2C_0}\right)^{-1} = \left(\frac{1}{C_0}\right)^{-1} = C_0.$$

The energy stored in all physical four capacitors is equal to energy stored in the equivalent capacitor charged to a potential difference of V_0,

$$\boxed{U = \frac{1}{2} C_0 V_0^2}$$

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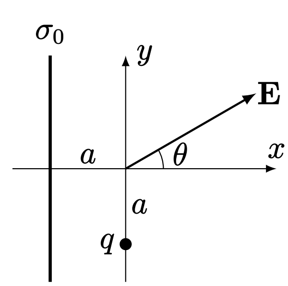
2) (25 points) A uniformly-charged infinite plane of surface charge density $\sigma_0 > 0$ is located in the plane $x = -a$. A particle of charge $q > 0$ is located at $(0, -a)$. At the origin, the electric field makes an angle θ with the positive x-axis, as shown. Simplifying your result so that it contains neither k nor ϵ_0, find σ_0.



Solution

At the origin, the x-component of the electric field is purely due to infinite plane, while the y-component of the electric field is purely due to the point charge. We have

$$E_x = \frac{\sigma_0}{2\epsilon_0} \quad \text{and} \quad E_y = \frac{1}{4\pi\epsilon_0}\frac{q}{a^2}$$

Then,

$$\tan\theta = \frac{E_y}{E_x} = \frac{1}{4\pi\epsilon_0}\frac{q}{a^2}\frac{2\epsilon_0}{\sigma_0} \implies \boxed{\sigma_0 = \frac{q}{2\pi a^2 \tan\theta}}$$

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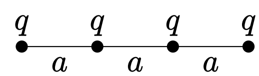
3) (25 points) Four particles, each of mass m and charge q, are arranged along a line, each spaced a distance a apart, as shown. We release the rightmost particle while holding the remaining three particles in place. What is the speed of this particle after it is has traveled a very far distance?



Solution This is an application of the conservation of mechanical energy,

$$U_0 + K_0 = U_1 + K_1$$

The initial potential energy is calculated by summing over all pairs of charges. There are four charges, so we know that, like the square charge distribution of MP02-Q10, we should find six pairings. Indeed, we find three pairs with distance a, two pairs with distance $2a$, and one pair with distance $3a$. Then,

$$U_0 = \frac{3kq^2}{a} + \frac{2kq^2}{2a} + \frac{kq^2}{3a} = \frac{13}{3}\frac{kq^2}{a}$$

When calculating the final potential energy, we need only consider the three stationary particles. With three charges, we should expect three pairings, like in the equilateral triangle of Example 2.10. Indeed, we find two pairs with distance a and one pair with distance $2a$. Then,

$$U_1 = \frac{2kq^2}{a} + \frac{kq^2}{2a} = \frac{5}{2}\frac{kq^2}{a}$$

Using $K_0 = 0$ and $K_1 = mv^2/2$, the conservation of mechanical energy is

$$U_0 = U_1 + K_1 \implies \frac{13}{3}\frac{kq^2}{a} = \frac{5}{2}\frac{kq^2}{a} + \frac{1}{2}mv^2 \implies \frac{11}{6}\frac{kq^2}{a} = \frac{1}{2}mv^2 \implies \boxed{v = \sqrt{\frac{11}{3}\frac{kq^2}{am}}}$$

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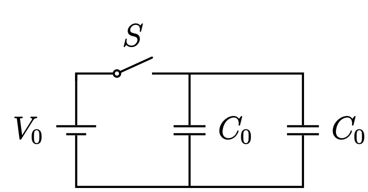
4) (25 points) Initially, switch S is closed so that the the battery of emf V_0 charges the two parallel-plate capacitors shown in the diagram. Then, the switch is opened and remains open. We then insert a dielectric κ into one of the capacitors, completely filling its gap. Finally, we increase the gap spacing of the other capacitor by a factor α. What is the final energy stored in the capacitors?



Solution The initial effective capacitance is $C_i = 2C_0$, and the net charge transferred to the two capacitors is

$$Q_0 = V_0 C_i = 2V_0 C_0.$$

After opening the switch, the charge on the effective capacitor remains Q_0. The final effective capacitance is

$$C_f = \kappa C_0 + \frac{C_0}{\alpha} = C_0\left(\kappa + \frac{1}{\alpha}\right)$$

Then,

$$U_f = \frac{Q_0^2}{2C_f} = \frac{4V_0^2 C_0^2}{2C_0(\kappa + 1/\alpha)} = \boxed{\frac{2C_0 V_0^2}{\kappa + 1/\alpha}}$$

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