

- Wait until instructed to begin.
- This exam is closed-book, with no external notes, no external scratch paper, and no electronic devices.
- Use the back of this coversheet for scratch work. If needed, extra scratch paper is available.
- If your work continues on the scratch page, then make a note in your solution.
- You may unstaple your exam, but please keep the pages in order and include this coversheet.
- Present your photo ID when you hand in your exam.

Electric field

$$k = \frac{1}{4\pi\epsilon_0} \quad |\mathbf{F}| = \frac{1}{4\pi\epsilon_0} \frac{|q_1q_2|}{r^2}$$

$$\mathbf{E}(x, y, z) = \frac{\mathbf{F}_q(x, y, z)}{q}$$

$$|\mathbf{E}| = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2} \quad |\mathbf{E}| = \frac{1}{4\pi\epsilon_0} \frac{|Q_{in}(r)|}{r^2}$$

$$\sigma = Q/A \quad |\mathbf{E}| = \frac{|\sigma|}{2\epsilon_0}$$

$$p = qd \quad \tau = pE \sin \phi$$

$$\mathbf{E}_{axis} \approx \frac{1}{4\pi\epsilon_0} \frac{2\mathbf{p}}{r^3} \quad \mathbf{E}_{plane} \approx -\frac{1}{4\pi\epsilon_0} \frac{\mathbf{p}}{r^3}$$

Capacitance

$$Q = VC \quad C_{plate} = \frac{\kappa\epsilon_0 A}{d}$$

$$U = \frac{Q^2}{2C} = \frac{QV}{2} = \frac{CV^2}{2} \quad u_E = \frac{1}{2}\kappa\epsilon_0 E^2$$

$$C_p = C_1 + C_2 + \dots \quad 1/C_s = 1/C_1 + 1/C_2 + \dots$$

$$E = E_0/\kappa \quad C = \kappa C_0$$

Mechanics

$$v = v_0 + at \quad x = x_0 + v_0t + at^2/2$$

$$v^2 = v_0^2 + 2a(x - x_0) \quad a_c = v^2/r$$

$$K = mv^2/2 \quad W = F_x \Delta x = -\Delta U$$

Electric potential

$$U = qV \quad E_s \approx -\Delta V/\Delta s$$

$$V = \sum_i \frac{1}{4\pi\epsilon_0} \frac{q_i}{r_i} \quad U = \frac{1}{4\pi\epsilon_0} \sum_{\text{all pairs}} \frac{q_i q_j}{r_{ij}}$$

$$V = -E_0 x \quad U = -qE_0 x$$

$$U = -pE \cos \phi \quad V = \frac{p \cos \theta}{4\pi\epsilon_0 r^2}$$

$$U_0 + K_0 = U_1 + K_1 \quad 0 = \sum_{\text{loop}} (\Delta V)_i$$

Miscellaneous

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \cos \theta = \frac{\text{adj}}{\text{hyp}} \quad \tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$a^2 + b^2 = c^2 \quad ; \quad a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$\text{Sphere: } A = 4\pi r^2 \quad V = (4/3)\pi r^3$$

$$\text{Circle: } C = 2\pi r \quad A = \pi r^2$$

$$ax^2 + bx + c = 0 \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Charge : C

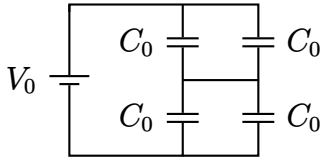
Electric field : N/C = V/m

Electric potential : V = J/C

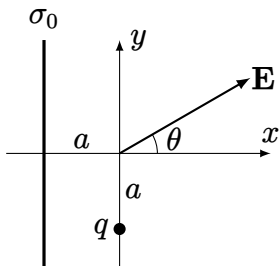
Capacitance : F = C/V

M = 10⁶, k = 10³, m = 10⁻³, μ = 10⁻⁶

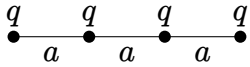
- 1) (25 points) Four capacitors, each of capacitance C_0 , and a battery of emf V_0 are connected in the circuit shown below. Find the total potential energy stored in the capacitors.



- 2) (25 points) A uniformly-charged infinite plane of surface charge density $\sigma_0 > 0$ is located in the plane $x = -a$. A particle of charge $q > 0$ is located at $(0, -a)$. At the origin, the electric field makes an angle θ with the positive x -axis, as shown. Simplifying your result so that it contains neither k nor ϵ_0 , find σ_0 .



- 3) (25 points) Four particles, each of mass m and charge q , are arranged along a line, each spaced a distance a apart, as shown. We release the rightmost particle while holding the remaining three particles in place. What is the speed of this particle after it has traveled a very far distance?



- 4) (25 points) Initially, switch S is closed so that the battery of emf V_0 charges the two parallel-plate capacitors shown in the diagram. Then, the switch is opened and remains open. We then insert a dielectric κ into one of the capacitors, completely filling its gap. Finally, we increase the gap spacing of the other capacitor by a factor α . What is the final energy stored in the capacitors?

