• Wait until instructed to begin.
• This exam is closed-book, with no external notes or scratch paper, and no electronic devices.
• Use this coversheet for scratch work. If needed, extra scratch paper is available.
• If your work continues on the scratch page, then make a note in your solution.
• You may unstaple your exam, but please keep the pages in order and include this coversheet.
• Have your photo ID available during the exam.
• Due to exam schedule conflicts, a few students will be taking this exam later this evening. Please do not discuss this exam until after 8AM Thursday.
Electric field

\[ k = \frac{1}{4\pi\epsilon_0} \]
\[ \mathbf{E}(x, y, z) = \frac{\mathbf{F}_d(x, y, z)}{q} \]
\[ |\mathbf{E}| = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2} \]
\[ \mathbf{E}_\text{axis} \approx \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3} \]
\[ \mathbf{E}_\text{plane} \approx -\frac{1}{4\pi\epsilon_0} \frac{p}{r^3} \]

Capacitance

\[ Q = VC \]
\[ C = \frac{\kappa\epsilon_0 A}{d} \]
\[ U = \frac{Q^2}{2C} = \frac{QV}{2} = \frac{CV^2}{2} \]
\[ U_{\text{plate}} = \frac{1}{2\kappa\epsilon_0} E^2 \]
\[ C_p = C_1 + C_2 + \cdots \]
\[ 1/C_s = 1/C_1 + 1/C_2 + \cdots \]
\[ E = E_0/k \]
\[ C = \kappa C_0 \]

Magnetic fields and forces

\[ \Delta B = \frac{\mu_0 I \Delta x \sin \theta}{2\pi} \]
\[ B_{\text{wire}} = \frac{\mu_0 I}{2\pi} \]
\[ B_{\text{loop}} = \frac{\mu_0 I}{2a} \]
\[ B_{\text{sol}} = \mu_0 n I \]
\[ n = N/L \]
\[ B_{\text{axis}} = \frac{\mu_0 2m}{4\pi d^3} \]
\[ m = m A \]
\[ \tau = mB \sin \theta \]
\[ U = -mB \cos \theta \]
\[ F = |q| vB \sin \alpha \]
\[ F = IlB \sin \alpha \]
\[ r = mv/(qB) \]
\[ f = qB/(2\pi m) \]

Electric potential

\[ U = qV \]
\[ E_s \approx -\Delta V/\Delta s \]
\[ V = \sum \frac{q_i}{4\pi\epsilon_0 r_i} \]
\[ U = \frac{1}{4\pi\epsilon_0} \sum \frac{q_i q_j}{r_{ij}} \]
\[ V = -E_0 x \]
\[ U = -qE_0 x \]
\[ U = -pE \cos \phi \]
\[ V = \frac{p \cos \theta}{4\pi\epsilon_0 r^2} \]
\[ U_0 + K_0 = U_1 + K_1 \]
\[ 0 = \sum_{\text{loop}} (\Delta V)_i \]

Current and resistance

\[ R = \frac{\rho L}{A} \]
\[ V = IR \]
\[ I_{\text{rms}} = I_0/\sqrt{2} \]
\[ V_{\text{rms}} = V_0/\sqrt{2} \]
\[ P = VI = V^2/R = I^2R \]
\[ P_{\text{avg}} = V_{\text{rms}} I_{\text{rms}} = V_{\text{rms}}^2/R = I_{\text{rms}}^2R \]

\[ \sum I_{\text{in}} = \sum I_{\text{out}} \]
\[ 0 = \sum \Delta V_i \]
\[ \tau = RC \]
\[ I(t) = I_0 e^{-t/\tau} \]
\[ V_d(t) = V_0 e^{-t/\tau} \]
\[ V_c(t) = V_0 (1 - e^{-t/\tau}) \]
\[ R_s = R_3 + R_2 + \cdots \]
\[ 1/R_p = 1/R_1 + 1/R_2 + \cdots \]

Mechanics

\[ v = v_0 + at \]
\[ x = x_0 + v_0 t + at^2/2 \]
\[ v^2 = v_0^2 + 2a(x - x_0) \]
\[ a_c = v^2/r \]
\[ K = m v^2/2 \]
\[ W = F_x \Delta x = -\Delta U \]
1) (10 points) Four charges are arranged at the corners of a square of side length $a\sqrt{2}$, as shown. Find the magnitude of the electric field at the center of the square, taking $q > 0$.

2) (10 points) A particle of positive charge $q$ and kinetic energy $K$ is traveling horizontally when it enters a region between two horizontal parallel plates, as shown. The plates, of length $L$, are separated by a gap $d$ and have a potential difference $\Delta V$. Upon exiting the gap, the particle makes an angle $\theta$ with respect to horizontal. Find an expression for $\theta$. 
3) (10 points) In the circuit below, $\mathcal{E}_0 = 3$ V, $C_1 = 6$ $\mu$F, $C_2 = 2$ $\mu$F, and $C_3 = 1$ $\mu$F. Find each capacitor’s potential difference, charge, and energy.
4) (10 points) Consider a spherical conducting shell of resistivity $\rho$, radius $r_1$, and shell thickness $d_1$. When we apply a potential difference between the shell’s inner and outer surfaces, a radial current flows. a) Assuming a thin shell thickness $d_1 \ll r_1$, find the electrical resistance $R_1$ to radial current flow through the shell. b) We uniformly stretch the sphere out to a larger radius $r_2$ with a thinner shell thickness $d_2$ so that the new resistance is $R_2$. Find an expression for $R_2/R_1$ in terms of $r_2$ and $r_1$.

5) (10 points) After installing the battery in the circuit below, we wait a long time for the capacitors to fully charge. Find the fully charged potential difference across each capacitor.

![Circuit Diagram](image)
6) (10 points) A current $4I_0$ flows along the $z$-axis and a current $3I_0$ flows along the $y$-axis, as shown. Find the magnitude of the resulting magnetic field at point $P$.

![Diagram of currents and point P](image)

7) (10 points) A slidewire is in nonuniform magnetic field $|\mathbf{B}| = \alpha x$, where $\alpha$ is a positive constant. You pull on the slidewire with constant velocity $v$, as shown. a) Find the magnitude and direction of the resulting current. b) Find the magnetic drag force and the mechanical power your hand must provide to maintain the velocity.

![Diagram of slidewire and magnetic field](image)
8) (10 points) A semipermeable membrane of thickness $d$ allows an ion species of positive charge $q$ to freely diffuse into and out of a cell while preventing all other species from passing. At temperature $T$, the ion concentrations are in equilibrium, and it is noted that, inside the membrane, there is a strong electric field $E_0$ directed inward, into the cell. 

a) Find an expression for $c_{\text{out}}/c_{\text{in}}$ in terms of the given quantities.

b) Is the ion concentration larger inside or outside the cell? Using physical reasoning, including a sketch, briefly explain why this is so.
9) (10 points) In a photoelectric demonstration lab, the maximum kinetic energy of photoelectrons is $K_0$. Reducing the wavelength of the incident light to half of its initial value increases the maximum photoelectron kinetic energy to $K_1$. a) What is the work function of the cathode? b) What was the initial wavelength?

10) (10 points) A particle of mass $m$ is in a box of length $L$. The system is in contact with a heat reservoir at temperature $T$. At what value of $T$ is the particle exactly twice as likely to be in the ground state rather than in the first excited state?