Physics 1C - Second Midterm

Thursday, May 25, 10-11:50AM

UCLA / Spring 2023 / Brian Naranjo

NAME ____________________

ID ____________________

- Wait until instructed to begin.
- This exam is closed-book, with no external notes, no external scratch paper, and no electronic devices.
- Use this coversheet for scratch work. If needed, extra scratch paper is available.
- This exam will be curved aggressively so that there will be roughly 30% As, 30% Bs, and 30% Cs. Grades will *only* be curved upward.
- The TAs and I will provide any requested mathematical identity.
- Paperclip your pages together, **in order**, including this coversheet on top.

Maxwell's Equations

$\oint_S \mathbf{E} \cdot d\mathbf{a} = \frac{Q_{\text{enc}}}{\epsilon_0}$	Gauss's law
$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt}$	Faraday's law
$\oint_S \mathbf{B} \cdot d\mathbf{a} = 0$	Gauss's law for $\mathbf{B}$
$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}} + \mu_0\epsilon_0 \frac{d\Phi_E}{dt}$	Ampère's law

Electrostatics

$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$	Force on charge
$\mathbf{E} = \frac{q\hat{\mathbf{R}}}{4\pi\epsilon_0 R^2}$	Coulomb's law
$d\mathbf{F} = I d\mathbf{l} \times \mathbf{B}$	Force on current
$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I' d\mathbf{r}' \times \hat{\mathbf{R}}}{R^2}$	Biot-Savart law

Electromagnetic waves

$I_d = \epsilon_0 \frac{d\Phi_E}{dt}$	Displacement current
$c = 1/\sqrt{\mu_0\epsilon_0}$	Speed of light
$\mathbf{S} = (1/\mu_0)\mathbf{E} \times \mathbf{B}$	Poynting vector
$u_E = (1/2)\epsilon_0 E^2$	Electric energy density
$u_B = B^2/(2\mu_0)$	Magnetic energy density
$v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{n}$	Speed of light in matter

Sinusoidal EM waves

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 \cos(\mathbf{k} \cdot \mathbf{r} - \omega t)$$
$$\mathbf{B}(\mathbf{r}, t) = \mathbf{B}_0 \cos(\mathbf{k} \cdot \mathbf{r} - \omega t)$$
$$\mathbf{E}_0 = c\mathbf{B}_0 \times \hat{\mathbf{k}}$$
$$\omega = 2\pi f, \quad k = 2\pi/\lambda, \quad \omega = ck, \quad \lambda f = c$$
$$\langle u \rangle = (1/2)\epsilon_0 E_0^2, \quad I \equiv \langle S \rangle = \sqrt{\epsilon_0/\mu_0} E_0^2/2$$
$$\langle p_{\text{rad}}^{\text{abs}} \rangle = I/c, \quad \langle p_{\text{rad}}^{\text{refl}} \rangle = 2I/c$$

Geometric optics

$\theta_i = \theta_r$	Specular reflection
$n_1 \sin\theta_1 = n_2 \sin\theta_2$	Snell's law
$f = r/2$	Spherical mirror
$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$	Thin lens equation
$m = -\frac{s'}{s}$	Lateral magnification
$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r}$	Refractive surface
$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$	Lensmaker's equation
$\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$	Free propagation
$\begin{pmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{pmatrix}$	Dielectric interface
$\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$	Thin lens
$\begin{pmatrix} m & 0 \\ -1/f & 1/m \end{pmatrix}$	Thin lens system

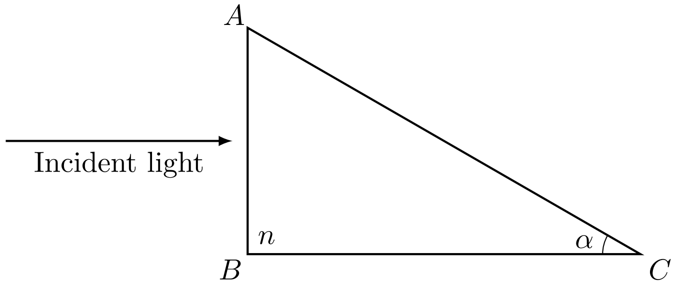
Interference

$\text{OPL} \equiv \int_C n(s)\, ds$	Optical path length
$\lambda_0 = n\lambda$	Vacuum wavelength
$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$	Two-beam intensity
$I = I_0 \cos^2(\phi/2)$	Two-beam intensity
$\phi(\mathbf{r}) = (2\pi/\lambda_0)\text{OPD} + \Delta\phi$	Two-beam phase
$nd \sin\theta = m\lambda_0$	Two-slit constructive
$nd \sin\theta = (m + 1/2)\lambda_0$	Two-slit destructive

Math

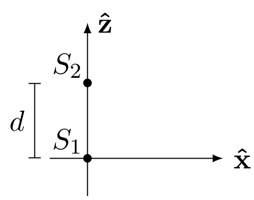
$$(1+x)^\alpha \approx 1 + \alpha x \quad (|x| \ll 1)$$
$$\langle A(t)B(t) \rangle = (1/2)\,\text{Re}(\widetilde{A}\widetilde{B}^*)$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

1) (10 points) A ray of light is incident normally on face AB of a glass prism of refractive index n, immersed in air. Find the largest value of the angle α so that the ray is totally internally reflected at face AC.

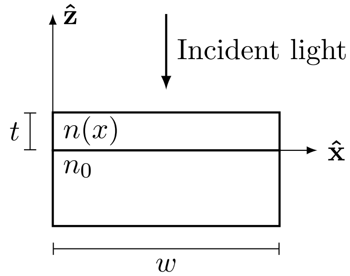


2) (10 points) An object, possibly either virtual or real, is located at $z = 0$. Find the focal length f and location z_L of a lens such that the resulting image, possibly either virtual or real, is located at $z' = d$ with a magnification of $m = +2$.

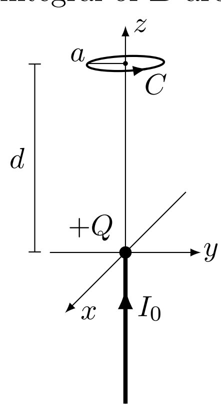
3) (10 points) Two coherent sources S_1 and S_2 of equal power emit at the same wavelength λ_0, but they are 180° out of phase with each other. Assuming $\lambda_0 \ll d$, find the largest finite value $x > 0$ on the x-axis for which there is complete destructive interference. Justify your approximation.



4) (15 points) Light of wavelength λ_0 is incident in air, from above, on a thin film of thickness $t = 2\lambda_0/3$ and width w placed on a substrate of refractive index $n_0 = 1.6$. If the the film's index of refraction varies over its width according to $n(x) = 1 + (x/w)$, find the *two* values of x for which the reflected light *constructively* interferes with the incident light.



5) (15 points) A current I_0 flows along the z axis, in the positive z direction, terminating at the origin, where a positive charge $Q(t) = I_0\, t$ accumulates. Loop C is a counter-clockwise circular path of radius a, centered on the z axis, in the plane $z = d$, as shown. Assuming $a \ll d$, find the line integral of $\mathbf{B}$ around loop C.

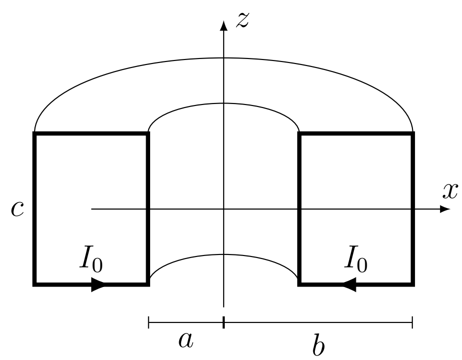


6) (20 points) A cutaway view of an ideal toroidal coil, whose axis-of-symmetry coincides with the z axis, is shown below. It has rectangular cross-section, inner radius a, outer radius b, height c, current I_0, and a total of N turns. There is an electric field $E = -E_0\hat{\mathbf{z}}$.

a) Find the Poynting vector $\mathbf{S}$ *everywhere.*

b) Calculate the flux of the Poynting vector through the toroid's inner surface, oriented in the *outward* direction,

$$\oint_S \mathbf{S} \cdot d\mathbf{a}$$



7) (20 points) An optical system consists of

- A converging lens of focal length $f > 0$, located at $z = 2f$.
- A concave mirror, with radius of curvature $R = 2f$ and vertex located at $z = 0$. It is oriented to the *right*, so that its center-of-curvature is at $z = 2f$.
- An upright arrow of height y_1, located in the object plane at $z = 0.5f$.

The arrow only emits light moving toward the mirror, so don't consider any light that initially moves toward the lens.

a) Calculate the location z_2' and total magnification m of the resulting image.

b) Sketch the system, including three principal rays for both the mirror and the lens. The mirror's third principal ray reflects off its vertex. Identify all objects and images as real or virtual using our usual notation (e.g., RO_1 for the real object of subsystem 1). Remember that something can be both image and an object.

c) A ray (y_1, θ_1) passes through the optical system, beginning in the object plane. Find the ray (y_2, θ_2) in the image plane.