Physics 1C - First Midterm

Thursday, April 27, 10-11:50AM

UCLA / Spring 2023 / Brian Naranjo

NAME ____________________

ID ____________________

- Wait until instructed to begin.
- This exam is closed-book, with no external notes, no external scratch paper, and no electronic devices.
- Use the back of this coversheet for scratch work. If needed, extra scratch paper is available.
- This exam will be curved aggressively so that there will be roughly 30% As, 30% Bs, and 30% Cs. Grades will *only* be curved upward.
- The TAs and I will provide any requested mathematical identity.
- Paperclip your pages together, **in order**, including this coversheet on top.

Maxwell's Equations

$\oint_S \mathbf{E} \cdot d\mathbf{a} = \frac{Q_{\text{enc}}}{\epsilon_0}$	Gauss's law
$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_{\text{enc}}}{dt}$	Faraday's law
$\oint_S \mathbf{B} \cdot d\mathbf{a} = 0$	Gauss's law for **B**
$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}$	Ampère's law

Magnetostatics

$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$	Force on charge
$d\mathbf{F} = I d\mathbf{l} \times \mathbf{B}$	Force on current
$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I' d\mathbf{r}' \times \hat{\mathbf{R}}}{R^2}$	Biot-Savart law
$\boldsymbol{\mu} = I\mathbf{A}$	Magnetic moment
$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$	Torque on $\boldsymbol{\mu}$
$U = -\boldsymbol{\mu} \cdot \mathbf{B}$	Energy of $\boldsymbol{\mu}$

RLC transients

$\tau_C = RC$	RC time constant
$\tau_L = L/R$	RL time constant
$\omega_0 = 1/\sqrt{LC}$	Resonant frequency
$\omega = \omega_0 \sqrt{1 - 1/(2\omega_0 \tau_L)^2}$	Damped frequency
$q(t) = Ae^{-t/(2\tau_L)} \cos(\omega t + \phi)$	Underdamped RLC

Induction

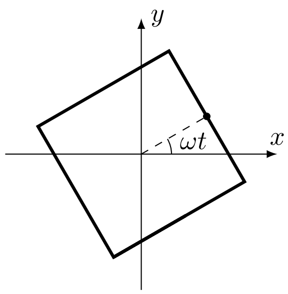
$\mathcal{E}_{ab} = \int_a^b \mathbf{f} \cdot d\mathbf{l}$	EMF
$\mathcal{E} = -\frac{d\Phi}{dt}$	Faraday's flux rule
$\Phi_{k \to j} = M_{jk} I_k$	Mutual inductance
$M_{jk} = M_{kj}$	Reciprocity
$\Phi = LI$	Self inductance
$Q = VC$	Capacitance
$U_B = LI^2/2$	Inductor energy
$U_E = Q^2/(2C)$	Capacitor energy
$u_B = B^2/(2\mu_0)$	Magnetic energy density

AC circuits

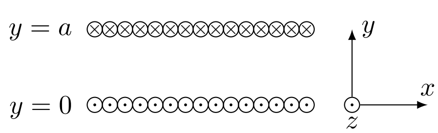
$Z_R = R$	Resistor
$Z_L = i\omega L$	Inductor
$Z_C = -i/(\omega C)$	Capacitor
$\widetilde{\mathcal{E}} = \mathcal{E}_0 e^{i\omega t}$	Phasor
$\widetilde{V} = \widetilde{I} Z$	AC Ohm's law
$\langle A(t) B(t) \rangle = (1/2)\,\text{Re}(\widetilde{A}\widetilde{B}^*)$	Time-average
$I_{\text{rms}} = \sqrt{\langle I^2(t) \rangle}$	RMS current
$\text{Re}(1/z) = \text{Re}(z)/\lvert z \rvert^2$	Useful identity

1) (10 points) A charge q is moving with velocity $\mathbf{v} = v_0 \cos\alpha\,\hat{\mathbf{x}} + v_0 \sin\alpha\,\hat{\mathbf{z}}$ in uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{x}}$. If we want the particle to maintain the same velocity, what electric field should we introduce?

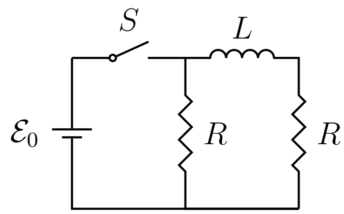
2) (10 points) A square loop of side length a and resistance R is centered on the origin and is rotating counterclockwise in the xy plane with angular velocity ω, as shown. If there is a magnetic field $\mathbf{B} = B_0 \cos\omega t\,\hat{\mathbf{z}}$, find the magnitude *and* direction of the loop's induced current.



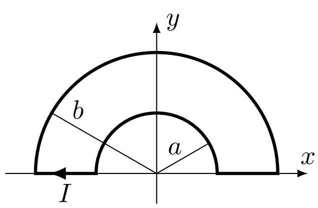
3) (10 points) Consider *two* infinite sheets of tightly packed wires. In the lower sheet, located in the plane $y = 0$, each wire is parallel to the z axis and carries current I in the positive z direction. In the upper sheet, located in the plane $y = a$, each wire is also parallel to z axis but instead carries current $2I$ in the *negative* z direction. Along the x direction, each sheet separately carries n wires per unit length. Find the magnetic field $\mathbf{B}$ everywhere.



4) (15 points) In the circuit below, switch S is closed shut for a very long time, so that the currents are steady. Then, at $t = 0$, the switch is abruptly opened. Find, by any method, the subsequent current $I_L(t)$ through the inductor, and indicate whether the current is flowing clockwise or counterclockwise.



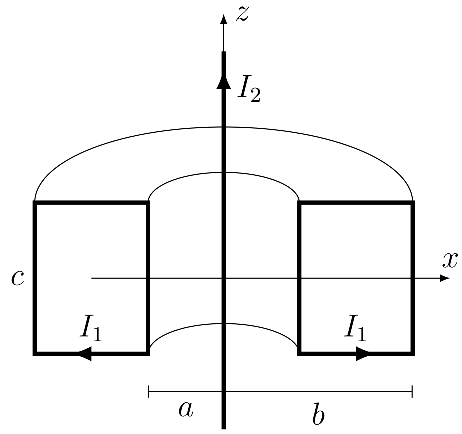
5) (15 points) A current I flows clockwise in the loop shown below. Using the Biot-Savart law, write a derivation for the magnetic field $\mathbf{B}$ at the origin.



6) (20 points) A cutaway view of an ideal toroidal coil, whose axis-of-symmetry coincides with the z axis, is shown below. It has rectangular cross-section, inner radius a, outer radius b, height c, and a total of N turns. In addition to the toroidal coil, there is a long straight wire that coincides with the z axis. The directions of *positive* currents I_1 and I_2 in these conductors are indicated with arrows

a) What is the mutual inductance M of the toroidal coil and the long straight wire?[1]

b) If we send a pulse of current through the toroidal coil, $I_1(t) = I_0 \exp[-t^2/(2\tau^2)]$, then, assuming that the long straight wire has total resistance R_2, find the current $I_2(t)$ induced in the straight wire.



[1]Remember that M can be either negative or positive!

7) (20 points) Consider a current source $I_0(t) = I_0 \cos \omega t$ driving the circuit below, where the positive sense of current is indicated with arrows. Find the real-valued amplitude I_L and phase ϕ_L such that the current through the inductor is written

$$I_L(t) = I_L \cos(\omega t - \phi_L).$$