Physics 1C - First Midterm
Thursday, April 27, 10-11:50AM
UCLA / Spring 2023 / Brian Naranjo

• Wait until instructed to begin.
• This exam is closed-book, with no external notes, no external scratch paper, and no electronic devices.
• Use the back of this coversheet for scratch work. If needed, extra scratch paper is available.
• This exam will be curved aggressively so that there will be roughly 30% As, 30% Bs, and 30% Cs. Grades will only be curved upward.
• The TAs and I will provide any requested mathematical identity.
• Paperclip your pages together, in order, including this coversheet on top.

Maxwell's Equations
\[ \oint_S \mathbf{E} \cdot d\mathbf{a} = \frac{Q_{\text{enc}}}{\varepsilon_0} \] Gauss's law
\[ \oint_C \mathbf{E} \cdot dl = -\frac{d\Phi_{\text{enc}}}{dt} \] Faraday's law
\[ \oint_S \mathbf{B} \cdot d\mathbf{a} = 0 \] Gauss's law for B
\[ \oint_C \mathbf{B} \cdot dl = \mu_0 I_{\text{enc}} \] Ampère's law

Induction
\[ \mathcal{E}_{ab} = \int_a^b \mathbf{f} \cdot dl \] EMF
\[ \mathcal{E} = -\frac{d\Phi}{dt} \] Faraday's flux rule
\[ \Phi_{k\rightarrow j} = M_{jk}I_k \] Mutual inductance
\[ M_{jk} = M_{kj} \] Reciprocity
\[ \Phi = LI \] Self inductance
\[ Q = VC \] Capacitance
\[ U_B = LI^2/2 \] Inductor energy
\[ U_E = Q^2/(2C) \] Capacitor energy
\[ u_B = B^2/(2\mu_0) \] Magnetic energy density

Magnetostatics
\[ \mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \] Force on charge
\[ d\mathbf{F} = I d\mathbf{l} \times \mathbf{B} \] Force on current
\[ d\mathbf{B} = \frac{\mu_0 I \, dl' \times \hat{R}}{4\pi R^2} \] Biot-Savart law
\[ \mu = I \mathbf{A} \] Magnetic moment
\[ \tau = \mu \times \mathbf{B} \] Torque on μ
\[ U = -\mu \cdot \mathbf{B} \] Energy of μ

AC circuits
\[ Z_R = R \] Resistor
\[ Z_L = i\omega L \] Inductor
\[ Z_C = -i/(\omega C) \] Capacitor
\[ \bar{\mathcal{E}} = \mathcal{E}_0 e^{i\omega t} \] Phasor
\[ \bar{V} = Z \] AC Ohm's law
\[ \langle A(t)B(t) \rangle = (1/2) \text{Re}(\bar{A}\bar{B}^*) \] Time-average
\[ I_{\text{rms}} = \sqrt{\langle I^2(t) \rangle} \] RMS current
\[ \text{Re}(1/z) = \text{Re}(z)/|z|^2 \] Useful identity

RLC transients
\[ \tau_C = RC \] RC time constant
\[ \tau_L = L/R \] RL time constant
\[ \omega_0 = 1/\sqrt{LC} \] Resonant frequency
\[ \omega = \omega_0 \sqrt{1 - 1/(2\omega_0 \tau_L)^2} \] Damped frequency
\[ q(t) = Ae^{-t/(2\tau_L)} \cos(\omega t + \phi) \] Underdamped RLC
1) (10 points) A charge $q$ is moving with velocity $\mathbf{v} = v_0 \cos \alpha \mathbf{\hat{x}} + v_0 \sin \alpha \mathbf{\hat{z}}$ in uniform magnetic field $\mathbf{B} = B_0 \mathbf{\hat{x}}$. If we want the particle to maintain the same velocity, what electric field should we introduce?

2) (10 points) A square loop of side length $a$ and resistance $R$ is centered on the origin and is rotating counterclockwise in the $xy$ plane with angular velocity $\omega$, as shown. If there is a magnetic field $\mathbf{B} = B_0 \cos \omega t \mathbf{\hat{z}}$, find the magnitude and direction of the loop’s induced current.

3) (10 points) Consider two infinite sheets of tightly packed wires. In the lower sheet, located in the plane $y = 0$, each wire is parallel to the $z$ axis and carries current $I$ in the positive $z$ direction. In the upper sheet, located in the plane $y = a$, each wire is also parallel to $z$ axis but instead carries current $2I$ in the negative $z$ direction. Along the $x$ direction, each sheet separately carries $n$ wires per unit length. Find the magnetic field $\mathbf{B}$ everywhere.
4) (15 points) In the circuit below, switch $S$ is closed shut for a very long time, so that the currents are steady. Then, at $t = 0$, the switch is abruptly opened. Find, by any method, the subsequent current $I_L(t)$ through the inductor, and indicate whether the current is flowing clockwise or counterclockwise.

![Circuit Diagram](image)

5) (15 points) A current $I$ flows clockwise in the loop shown below. Using the Biot-Savart law, write a derivation for the magnetic field $\mathbf{B}$ at the origin.

![Loop Diagram](image)
6) (20 points) A cutaway view of an ideal toroidal coil, whose axis-of-symmetry coincides with the
z axis, is shown below. It has rectangular cross-section, inner radius $a$, outer radius $b$, height $c$, and a total of $N$ turns. In addition to the toroidal coil, there is a long straight wire that coincides with the $z$ axis. The directions of positive currents $I_1$ and $I_2$ in these conductors are indicated with arrows.

a) What is the mutual inductance $M$ of the toroidal coil and the long straight wire?

b) If we send a pulse of current through the toroidal coil, $I_1(t) = I_0 \exp\left[-t^2/(2\tau^2)\right]$, then, assuming that the long straight wire has total resistance $R_2$, find the current $I_2(t)$ induced in the straight wire.

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\[M\] can be either negative or positive!

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\[\text{Remember that } M \text{ can be either negative or positive!}\]
7) (20 points) Consider a current source $I_0(t) = I_0 \cos \omega t$ driving the circuit below, where the positive sense of current is indicated with arrows. Find the real-valued amplitude $I_L$ and phase $\phi_L$ such that the current through the inductor is written

$$I_L(t) = I_L \cos(\omega t - \phi_L).$$