Physics 1C - Final

Tuesday, June 13, 11:30AM-2:30PM

UCLA / Spring 2023 / Brian Naranjo

NAME ____________________

ID ____________________

- Wait until instructed to begin.
- This exam is closed-book, with no external notes, no external scratch paper, and no electronic devices.
- Use this coversheet for scratch work. If needed, extra scratch paper is available.
- This exam will be curved aggressively so that there will be roughly 30% As, 30% Bs, and 30% Cs. Grades will *only* be curved upward.
- The TAs and I will provide any requested mathematical identity.
- Paperclip your pages together, **in order**, including this coversheet on top.

Maxwell's Equations

$\oint_S \mathbf{E}\cdot d\mathbf{a} = \frac{Q_{\text{enc}}}{\epsilon_0}$	Gauss's law
$\oint_C \mathbf{E}\cdot d\mathbf{l} = -\frac{d\Phi_B}{dt}$	Faraday's law
$\oint_S \mathbf{B}\cdot d\mathbf{a} = 0$	Gauss's law for $\mathbf{B}$
$\oint_C \mathbf{B}\cdot d\mathbf{l} = \mu_0 I_{\text{enc}} + \mu_0\epsilon_0\frac{d\Phi_E}{dt}$	Ampère's law

Magnetostatics

$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$	Force on charge
$d\mathbf{F} = I d\mathbf{r}\times\mathbf{B}$	Force on current
$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I' d\mathbf{r}'\times\hat{\mathbf{R}}}{R^2}$	Biot-Savart law
$\boldsymbol{\mu} = I\mathbf{A}$	Magnetic moment
$\boldsymbol{\tau} = \boldsymbol{\mu}\times\mathbf{B}$	Torque on $\boldsymbol{\mu}$
$U = -\boldsymbol{\mu}\cdot\mathbf{B}$	Energy of $\boldsymbol{\mu}$

RLC transients

$\tau_C = RC$	RC time constant
$\tau_L = L/R$	RL time constant
$\omega_0 = 1/\sqrt{LC}$	Resonant frequency
$\omega = \omega_0\sqrt{1 - 1/(2\omega_0\tau_L)^2}$	Damped frequency
$q(t) = Ae^{-t/(2\tau_L)}\cos(\omega t + \phi)$	Underdamped RLC

Geometric optics

$\theta_i = \theta_r$	Specular reflection
$n_1\sin\theta_1 = n_2\sin\theta_2$	Snell's law
$f = r/2$	Spherical mirror
$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$	Thin lens equation
$m = -\frac{s'}{s}$	Lateral magnification
$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r}$	Refractive surface
$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$	Lensmaker's equation
$\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$	Free propagation
$\begin{pmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{pmatrix}$	Dielectric interface
$\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$	Thin lens
$\begin{pmatrix} m & 0 \\ -1/f & 1/m \end{pmatrix}$	Thin lens system

Induction

$\mathcal{E}_{ab} = \int_a^b \mathbf{f}\cdot d\mathbf{l}$	EMF
$\mathcal{E} = -\frac{d\Phi}{dt}$	Faraday's flux rule
$\Phi_{k\to j} = M_{jk}I_k$	Mutual inductance
$M_{jk} = M_{kj}$	Reciprocity
$\Phi = LI$	Self inductance
$Q = VC$	Capacitance
$U_B = LI^2/2$	Inductor energy
$U_E = Q^2/(2C)$	Capacitor energy

AC circuits

$Z_R = R$	Resistor
$Z_L = i\omega L$	Inductor
$Z_C = -i/(\omega C)$	Capacitor
$\widetilde{\mathcal{E}} = \mathcal{E}_0 e^{i\omega t}$	Phasor
$\widetilde{V} = \widetilde{I}Z$	AC Ohm's law
$\langle A(t)B(t)\rangle = (1/2)\,\text{Re}(\widetilde{A}\widetilde{B}^*)$	Time-average
$I_{\text{rms}} = \sqrt{\langle I^2(t)\rangle}$	RMS current
$\text{Re}(1/z) = \text{Re}(z)/\lvert z\rvert^2$	Useful identity

Electromagnetic waves

$I_d = \epsilon_0\frac{d\Phi_E}{dt}$	Displacement current
$c = 1/\sqrt{\mu_0\epsilon_0}$	Speed of light
$\mathbf{S} = (1/\mu_0)\mathbf{E}\times\mathbf{B}$	Poynting vector
$u_E = (1/2)\epsilon_0 E^2$	Electric energy density
$u_B = B^2/(2\mu_0)$	Magnetic energy density
$v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{n}$	Speed of light in matter

Sinusoidal EM waves

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0\cos(\mathbf{k}\cdot\mathbf{r} - \omega t)$$
$$\mathbf{B}(\mathbf{r},t) = \mathbf{B}_0\cos(\mathbf{k}\cdot\mathbf{r} - \omega t)$$
$$\mathbf{E}_0 = c\mathbf{B}_0\times\hat{\mathbf{k}}$$
$$\omega = 2\pi f,\quad k = 2\pi/\lambda,\quad \omega = ck,\quad \lambda f = c$$
$$\langle u\rangle = (1/2)\epsilon_0 E_0^2,\quad I \equiv \langle S\rangle = \sqrt{\epsilon_0/\mu_0}E_0^2/2$$
$$\langle p_{\text{rad}}^{\text{abs}}\rangle = I/c,\quad \langle p_{\text{rad}}^{\text{refl}}\rangle = 2I/c$$

Interference

$\text{OPL} \equiv \int_C n(s)\,ds$	Optical path length
$\lambda_0 = n\lambda$	Vacuum wavelength
$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$	Two-beam intensity
$I = I_0\cos^2(\phi/2)$	Two-beam intensity
$\phi(\mathbf{r}) = (2\pi/\lambda_0)\text{OPD} + \Delta\phi$	Two-beam phase
$nd\sin\theta = m\lambda_0$	Two-slit constructive
$nd\sin\theta = (m+1/2)\lambda_0$	Two-slit destructive

Diffraction

$I = I_0\dfrac{\sin^2(N\phi/2)}{\sin^2(\phi/2)}$	N-beam interference
$\phi = (2\pi/\lambda_0)d\sin\theta$	N-slit phase increment
$I = I_0\,\text{sinc}^2(\beta/2)$	Finite slit diffraction
$\beta = (2\pi/\lambda_0)a\sin\theta$	Finite slit OPD
$I = I_0\dfrac{\sin^2(N\phi/2)}{\sin^2(\phi/2)}\,\text{sinc}^2(\beta/2)$	N finite slits

Math

$$(1+x)^\alpha \approx 1 + \alpha x \quad (|x| \ll 1)$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A \sin B$$

$$\text{sinc}\,x = (\sin x)/x$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Relativity

$$\beta = v/c$$

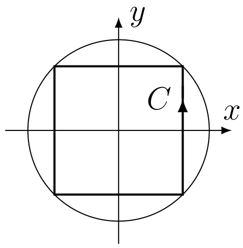
$$\gamma = 1/\sqrt{1-\beta^2}$$

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$$

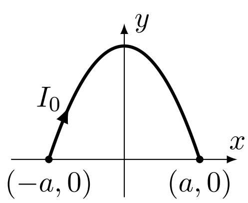
$$\begin{pmatrix} ct \\ x \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} ct' \\ x' \end{pmatrix}$$

Δt_0	Proper time
Δx_0	Proper length
$\Delta t = \gamma\Delta t_0$	Time dilation
$\Delta x = (\Delta x_0)/\gamma$	Length contraction
$\beta = \dfrac{\beta_1+\beta_2}{1+\beta_1\beta_2}$	Velocity addition
$E = \gamma mc^2$	Energy
$\mathbf{p} = \gamma m\mathbf{v}$	Momentum
$\mathbf{F} = d\mathbf{p}/dt$	Force
$E^2 = (mc^2)^2 + (pc)^2$	

1) (8 points) Consider a thick cylindrical wire of radius a whose axis coincides with the z axis. Uniformly over its cross section, it carries a current I_0 in the positive z direction, generating an azimuthal magnetic field $\mathbf{B}$. In the xy plane, path C is a square inscribed in the wire's cross section, oriented as shown. Find the line integral of the magnetic field around path C.



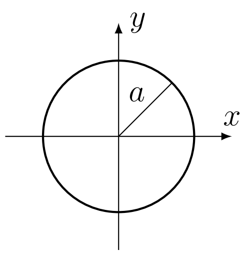
2) (8 points) A wire in the xy plane carries current I_0 from its initial point at $(-a, 0)$ to its final point at $(a, 0)$. The wire is bent into a parabola satisfying $y = y_0\,[1 - (x/a)^2]$, as shown below. Find the net force on the wire if it is immersed in a magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$.



3) (8 points) Alice and Bob rocket toward each other at a very high speed. From Alice's frame, Bob approaches at a speed equal to $(4/5)c$. Likewise, from Bob's frame, Alice approaches at a speed equal to $(4/5)c$. Meanwhile, you are stationed between Alice and Bob, where you note that Alice and Bob are both approaching you at the *same* speed. In terms of c, what is that speed?

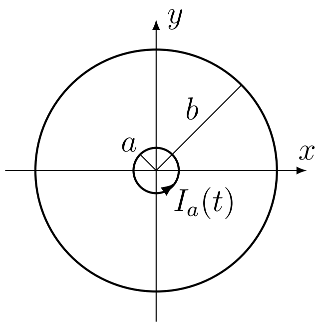
4) (10 points) A double slit is uniformly and coherently illuminated by light of wavelength λ_0. The slits are spaced a distance $d = 15\lambda_0$ apart and the width of each slit is $a = 3\lambda_0$. How many interference maxima lie within the central diffraction envelope?

5) (10 points) Consider a volume V of a metal with electrical conductivity σ. We draw the metal out into a uniformly thin wire formed into a circular loop of radius a in the xy plane, as shown. If the wire is then immersed in a time-varying magnetic field $\mathbf{B}(t) = \hat{\mathbf{z}} B_0 t/\tau$, find the magnitude and the direction of the induced current flowing in the circular loop.[1]



[1]Recall that a wire of length L, cross-section area A, and electrical conductivity σ has resistance $R = L/(\sigma A)$.

6) (12 points) Consider two circular conducting loops in the xy plane, centered on the origin. The inner loop has radius a, and the outer loop has radius b. You may assume that $a \ll b$. A time varying current $I_a(t) = I_0 \cos \omega t$ flows through the inner loop, where we take the positive sense of current to be in the counterclockwise direction. If R is the resistance of the outer loop, then find the magnitude and direction of current $I_b(t)$ induced in the outer loop.[2]

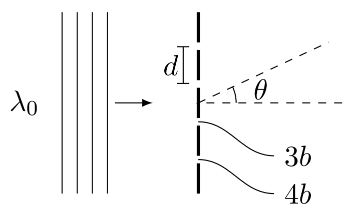


[2] Hmmm... maybe use the reciprocity of mutual inductance?

7) (12 points) Consider four thin slits that are uniformly spaced a distance d apart. The slits are coherently and uniformly illuminated by light of wavelength λ_0. We then cover each slit with a thin dielectric plate of refractive index n. The first plate's thickness is b, the second plate's thickness is $2b$, etc. The resulting intensity is observed at angle θ, as shown below.

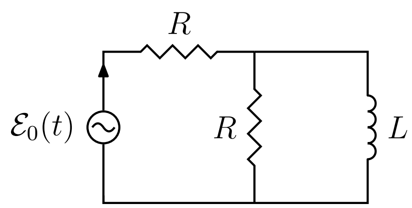
a) At what angle is the beam's central maximum θ_0?

b) What is the angular separation $\Delta\theta_0$ between the central maximum and either one of its adjacent minima?



8) (16 points) Consider a voltage source $\mathcal{E}_0(t) = \mathcal{E}_0 \cos \omega t$ driving the circuit below, where the positive sense of current is indicated with an arrow. Assuming the inductor's top terminal is its positive reference, find the real-valued amplitude V_L and phase ϕ_L such that the voltage across the inductor is written

$$V_L(t) = V_L \cos(\omega t + \phi_L).$$



9) (16 points) An optical system consists of

- A converging lens of focal length $f_1 = f > 0$, located at $z = 0$.
- A diverging lens of focal length $f_2 = -2f < 0$, located at $z = 3f$.
- An upright arrow of height y_1, located in the object plane at $z_1 = -f/2$.

a) Calculate the location z_2' and total magnification m of the resulting image.
b) Sketch the system, including three principal rays for each lens. Identify all objects and images as real or virtual using our usual notation (e.g., RO_1 for the real object of subsystem 1). Remember that something can be both an image and an object.
c) A ray (y_1, θ_1) passes through the optical system, beginning in the object plane. Find the ray (y_2, θ_2) in the image plane.