Physics 110B - Midterm 2

Monday, March 7, 10-11AM, in PAB 1434A

UCLA / Winter 2022 / Brian Naranjo

NAME _____

ID _____

- You may use a single letter-sized paper with handwritten notes, front and back.
- I will provide any requested mathematical identity.
- No electronic devices.
- Paperclip your pages together, with the white coversheet on top. Write one solution per page(s).
- This test will be curved so that at least 30% of you will receive an A. It is written to give a broad distribution of scores, so please don't despair if it is difficult.

Useful boundary condition identities:

$$\hat{\mathbf{n}} \cdot (\mathbf{F}_2 - \mathbf{F}_1) = \lim_{h \to 0} (h \nabla \cdot \mathbf{F}) \quad \text{and} \quad \hat{\mathbf{n}} \times (\mathbf{F}_2 - \mathbf{F}_1) = \lim_{h \to 0} (h \nabla \times \mathbf{F}),$$

where $\hat{\mathbf{n}}$ is the boundary normal pointing from medium 1 into medium 2.

Problems

1) (10 points) The region $z > 0$ is vacuum and the region $z < 0$ is filled with a perfect conductor. Traveling through the vacuum, an electromagnetic plane wave is normally incident on the boundary. The wave has linear polarization along $\hat{\mathbf{x}}$, electric field amplitude E_0, and angular frequency ω. Find the induced surface current on the boundary.

2) (10 points) A rectangular hollow waveguide has walls at $x = 0$, $x = a$, $y = 0$, and $y = b$, where $a > b$. For a TE_{10} mode propagating in the positive z direction with angular frequency ω, find the surface current density $\mathbf{K}(y, z, t)$ on the $x = 0$ wall and the surface charge density $\sigma(x, z, t)$ on the $y = 0$ wall.

3) (10 points) A sphere of radius a, centered on the origin, has a time-dependent but spatially uniform charge density

$$\rho(t) = \alpha t \qquad (-\infty < t < \infty)$$

Don't worry about the current that would be needed to establish such a charge density. Find the retarded scalar potential $\phi(r, t)$ for any distance r from the origin.

4) (10 points) There is a wire along the negative x-axis extending from $x = -\infty$ to the origin. It carries a current

$$I(t) = \begin{cases} 2\alpha t & t > 0 \\ 0 & t < 0 \end{cases}$$

such that, for $t > 0$, charge $q(t) = \alpha t^2$ accumulates at the origin. Find the retarded scalar potential $\phi(x, t)$ and the retarded vector potential $\mathbf{A}(x, t)$ on the positive x-axis ($x > 0$) for all time t.