Physics 110B - Final

Thursday, March 17, 8-11AM, in PAB 1434A

UCLA / Winter 2022 / Brian Naranjo

NAME ________________

ID ________________

- You may use a single letter-sized paper with handwritten notes, front and back.
- I will provide any requested mathematical identity.
- No electronic devices.
- Paperclip your pages together, with the white coversheet on top. Write one solution per page(s).
- This test will be curved so that at least 30% of you will receive an A.

Useful boundary condition identities:

$$\hat{\mathbf{n}} \cdot (\mathbf{F}_2 - \mathbf{F}_1) = \lim_{h \to 0} (h \nabla \cdot \mathbf{F}) \quad \text{and} \quad \hat{\mathbf{n}} \times (\mathbf{F}_2 - \mathbf{F}_1) = \lim_{h \to 0} (h \nabla \times \mathbf{F}),$$

where $\hat{\mathbf{n}}$ is the boundary normal pointing from medium 1 into medium 2.

The divergence in cylindrical coordinates is

$$\nabla \cdot \mathbf{A} = \frac{1}{s}\frac{\partial (s A_s)}{\partial s} + \frac{1}{s}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

Problems

1) (10 points) The axis of an infinitely long ideal solenoid coincides with the z-axis. It has radius a, turns per unit length n, and a time-varying current

$$I(t) = I_0 \cos \omega t.$$

Working in the quasistatic approximation and assuming that the positive direction of the current is counterclockwise as viewed from the positive z-axis looking towards the origin, find the electric field everywhere.

2) (10 points) There is a wire along the negative x-axis extending from $x = -\infty$ to the origin. It carries a current $I(t) > 0$ such that charge accumulates at the origin. Working in the quasistatic approximation, find the displacement current density's *outward* flux through a sphere of radius a centered on the origin.

3) (10 points) A spherical shell of radius a, uniform surface charge density, and total charge Q, is centered on the origin and split, about the xy plane, into two halves. The $z < 0$ lower half is fixed in position, while the $z > 0$ upper half is free to move. The two halves repel each other, so that you need to push down on the upper half. Using the Maxwell stress tensor, what is the force required to keep the upper half in place?

4) (10 points) The region between two concentric metal cylindrical shells, of radii a and b, with $a < b$, is filled with a linear conductive material whose conductivity has a radial dependence $\sigma(s) = \sigma_0 a/s$. If the outer shell is held at potential zero, while the inner shell is held at potential V_0, then what is the charge density $\rho(s)$ in the region between the two cylinders?

MORE PROBLEMS $\longrightarrow$

5) (10 points) The region $z > 0$ is vacuum and the region $z < 0$ is filled with a perfect conductor. Traveling through the vacuum, there is an electromagnetic plane wave of angular frequency ω and electric field

$$\mathbf{E}_I = \hat{\mathbf{x}} E_0 \cos(\mathbf{k}_I \cdot \mathbf{r} - \omega t)$$

where the incident wavevector is in the yz plane,

$$\mathbf{k}_I = k\,\hat{\mathbf{k}}_I = k(\hat{\mathbf{y}}\,\sin\theta - \hat{\mathbf{z}}\,\cos\theta).$$

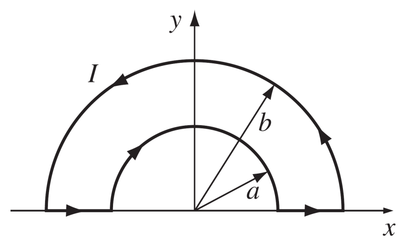
Find the induced surface current $\mathbf{K}(y,t)$.

6) (10 points) The region $|x| < a$ is vacuum. Elsewhere, $|x| > a$, is perfect conductor. In the vacuum channel, a TEM wave propagates in the positive z direction with angular frequency ω. On the wall at $x = a$, find the surface charge density $\sigma(z,t)$ and the surface current density $\mathbf{K}(z,t)$. Express your result with real values and, apart from an arbitrary overall amplitude and phase, express any variables you introduce in terms of constants or given values.

7) (10 points) A current

$$I(t) = \begin{cases} I_0 & t > 0 \\ 0 & t < 0 \end{cases}$$

flows through a circuit as shown below. At the origin, find the retarded vector potential $\mathbf{A}(t)$ and electric field $\mathbf{E}(t)$ for all time t.



8) (10 points) A sphere of radius b is centered on the origin and spinning about the x-axis with angular frequency ω. Viewed from the positive x-axis, it is spinning counter-clockwise. Half of the sphere has positive uniform charge density and net charge Q, while the other half of the sphere has charge density zero, so that the charged region is hemispherical. At $t = 0$, the rotating sphere is aligned so that the charged region is $z > 0$. In the farfield of the dipole approximation, assuming $b \ll 2\pi c/\omega \ll r$, find the *instantaneous* fields, $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$, at **both** $\mathbf{r} = x\,\hat{\mathbf{x}}$ and $\mathbf{r} = y\,\hat{\mathbf{y}}$.

Notes for Problem 8: In the farfield of the dipole approximation, the instantaneous electromagnetic fields are

$$\mathbf{E}(\mathbf{r},t) \approx \frac{\mu_0}{4\pi r}\left[(\hat{\mathbf{r}}\cdot\ddot{\mathbf{p}})\hat{\mathbf{r}} - \ddot{\mathbf{p}}\right] = \frac{\mu_0}{4\pi r}\left[\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\ddot{\mathbf{p}})\right]$$

$$\mathbf{B}(\mathbf{r},t) = \frac{1}{c}\,\hat{\mathbf{r}}\times\mathbf{E}(\mathbf{r},t),$$

where $\ddot{\mathbf{p}}$ is evaluated at $t_r = t - r/c$ and

$$\mathbf{p}(t) \equiv \int \mathbf{r}\rho(\mathbf{r},t)\,dV$$